Question 5 (25 marks)

(a) The *Sieve of Sundaram* is an infinite table of arithmetic sequences.
The terms in the first 4 rows and the first 4 columns of the table are shown below.

4	7	10	13		
7	12	17	22		
10	17	24	31		
13	22	31	40		

(i) Find the **difference** between the **sums** of the first 45 terms in the first two rows.

(ii) Find the number which is in the 60th row and 70th column of the table.

(b) The first two terms of a sequence are $a_1 = 4$ and $a_2 = 2$.
The general term is defined by $a_n = a_{n-1} - a_{n-2}$, when $n \geq 3$.
Write out the next 6 terms of the sequence **and hence** find the value of a_{2019}

Q5	Model Solution – 25 Marks	Marking Notes
(a) (i)	$$row\ 2: S_{45} = \frac{45}{2}[14 + 44(5)] = 5265$$ $$row\ 1: S_{45} = \frac{45}{2}[8 + 44(3)] = 3150$$ $$\therefore \text{Difference} = 2115$$	**Scale 10C (0, 4, 8, 10)** *Low Partial Credit:* Formulates S_{45} for row 1 or row 2 3+5+7 … Identifies *a* or *r* for either row 1 or row 2 *High Partial Credit:* S_{45} found for row 1 or row 2 *Full credit −1:* Fails to subtract
(a) (ii)	$$T_1(\text{in row } 60): T_{60} = 4 + (60 - 1)3 = 181$$ $$T_2 \text{ (in row } 60) = T_{60} \text{ of } 7, 12, 17, 22 \dots$$ $$T_{60} = 7 + (60 - 1)5 = 302$$ $$\therefore T_{70} \text{ of } 181, 302, \dots \dots$$ $$= 181 + (70 - 1)121 = 8530$$	**Scale 10D (0, 3, 5, 8, 10)** *Low Partial Credit:* Identifies T_{60} in column 1 or T_{70} In row 1 or equivalent Some relevant substitution into correct formula Identifies *a* or *d* for either row 1 or row 2 *Mid Partial Credit:* Finds *a* in row 60 or row 70 Finds *d* in row 60 or row 70 *High Partial Credit:* Formulates substituted expression for T_{70} in row 60 or T_{60} in column 70 Finds both *a* and *d* in row 60 or row 70
(b)	$$a_3 = a_2 - a_1 = 2 - 4 = -2$$ $$a_4 = a_3 - a_2 = -2 - 2 = -4$$ $$a_5 = a_4 - a_3 = -4 - (-2) = -2$$ $$a_6 = a_5 - a_4 = -2 - (-4) = 2$$ $$a_7 = a_6 - a_5 = 2 - (-2) = 4$$ $$a_8 = a_7 - a_6 = 4 - 2 = 2$$ Therefore, the sequence consists of a repeating pattern of $$4,\ 2, -2, -4, -2,\ 2$$ $$\therefore a_{2016} = 2 \text{ (multiple of 6)}$$ $$\Rightarrow a_{2019} = -2$$	**Scale 5C (0, 3, 4, 5)** *Low Partial Credit:* $a_3 = -2$ $a_3 = a_2 - a_1$ or similar *High Partial Credit:* Any 4 from a_3, a_4, a_5, a_6, a_7 and a_8 found *Full credit −1:* $a_3, a_4, a_5, a_6,$ and a_{2019} found