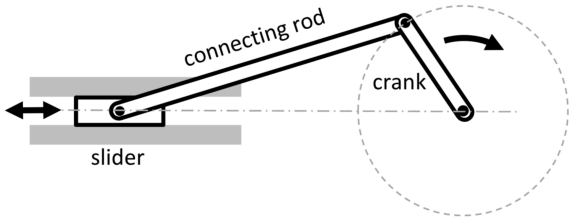
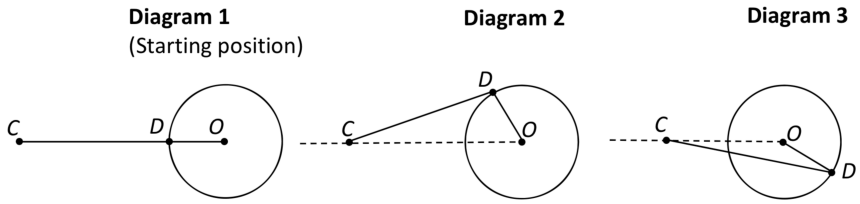
Question 9 **(40 marks)**

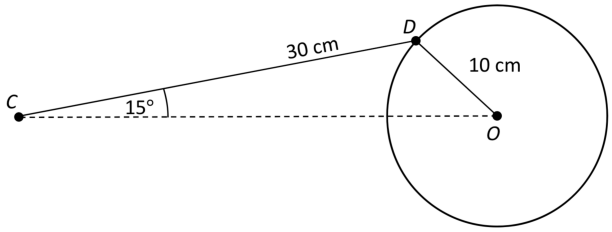
In engineering, a crank-and-slider mechanism can be used to change circular motion into motion back and forth in a straight line.



In the diagrams below, the crank $[OD]$ rotates about the fixed point O. The point C slides back and forth in a horizontal line. $[CD]$ is the rod that connects C to the crank. The diagrams below show three of the possible positions for C and D. $|OD| = 10$ cm and $|DC| = 30$ cm.



(a) The diagram shows a particular position of the mechanism with $|\angle DCO| = 15°$. Find $|\angle COD|$, correct to the nearest degree.



(b) As D moves in a circle around O, the angle α in the diagram below increases. The distance $|CX|$ can be considered to be a function of α and written as $f(\alpha)$.

(i) Write down the period and range of f.

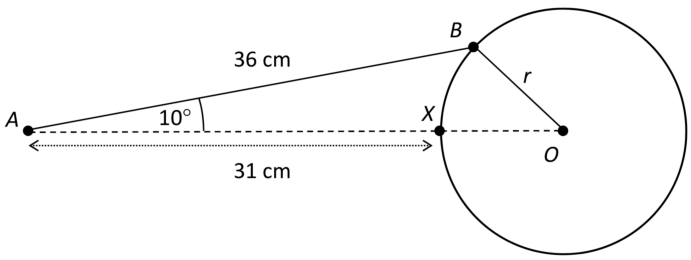
(ii) Complete the table below for $f(\alpha)$.
Give your answers correct to 2 decimal places where appropriate.
(**Note: Diagram 1** at the start of this question represents $\alpha = 0°$).

α	$0°$	$90°$	$180°$	$270°$	$360°$
$f(\alpha)$ (cm)	30				

(iii) Use your values from the table to draw a rough sketch of f in the domain $0° \leq \alpha \leq 360°$.

(iv) Referring to **Diagrams 1, 2, and 3** near the start of this question, for which of the three positions of the mechanism will a 1 degree change in α cause the greatest change in the position of C? Explain your answer.

(c) The diagramshows another crank-and-slider mechanism with different dimensions. In the diagram, $|AB| = 36$ cm, $|AX| = 31$ cm, and $|\angle BAO| = 10°$.
(Note: $|\angle OBA| = 90°$)
Find r, the length of the crank. Give your answer in cm, correct to the nearest cm.

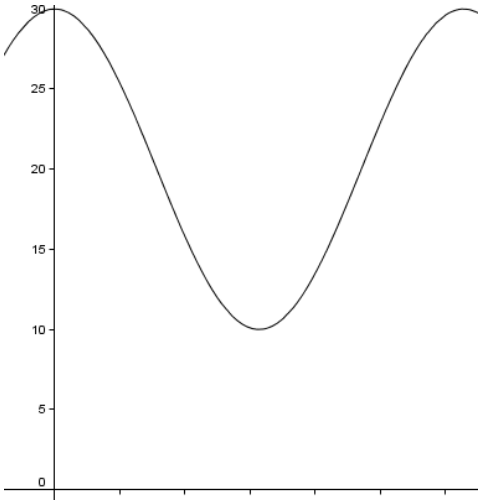


<table>
<tr><th>Q9</th><th>Model Solution – 40 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>$$\frac{10}{\sin 15} = \frac{30}{\sin x}$$
$$\sin x = \frac{30\ \sin 15}{10}$$
$$\sin x = 0{\cdot}77645$$
$$x = 51°$$</td><td>Scale 10C (0, 3, 7, 10)
Low Partial Credit:
Sine rule formulated with some substitution

High Partial Credit:
$\sin x$</td></tr>
<tr><td>(b)
(i)</td><td>period = 2π Range = [10, 30]</td><td>Scale 5C (0, 2, 4, 5)
Low Partial Credit:
Period or range correct

High Partial Credit:
Period correct and range partly correct
Period and range in incorrect order</td></tr>
<tr><td>(b)
(ii)</td><td><table><tr><td>α</td><td>$0°$</td><td>$90°$</td><td>$180°$</td><td>$270°$</td><td>$360°$</td></tr><tr><td>$f(\alpha)$
(cm)</td><td>30</td><td>18·28</td><td>10</td><td>18·28</td><td>30</td></tr></table></td><td></td></tr>
<tr><td></td><td></td><td>Scale 5C (0, 2, 4, 5)
Low Partial Credit:
1 correct new value

High Partial Credit:
2 correct new values</td></tr>
</table>

Q9	Marking Notes	
(b) (iii)		**Scale 10C (0, 3, 7, 10)** *Low Partial Credit:* 1 point from table plotted *High Partial Credit:* 3 points from table plotted
(b) (iv)	Answer: diagram 2 refer to the steepness of their graph at the three corresponding points **or** rely on the original geometry of the situation: the closer ∠CDO is to a right angle the more the connecting rod will get pulled or pushed by a small change in the crank angle	**Scale 5B (0, 2, 5)** *Partial Credit:* Diagram 2 identified but without reason or with invalid reason

(c)	$r^2 = 36^2 + (31 + r)^2$ $- 2(36)(31 + r)\cos 10°$ $r^2 = 1296 + 961 + 62r + r^2$ $-(2232\cos 10° - 72r\cos 10°)$ $8{\cdot}906r = 58{\cdot}91$ $r = 6{\cdot}62$ $r = 7$ **Or** $\lvert BX\rvert^2 = 36^2 + 31^2 - 2 \times 36 \times 31\cos 10°$ $\lvert BX\rvert^2 = 58{\cdot}91$ $\lvert BX\rvert = 7{\cdot}675$ $\frac{\sin 10°}{7{\cdot}675} = \frac{\sin \angle BXA}{36}$ $\angle BXA = 125{\cdot}462° \Rightarrow \angle BXO = 54{\cdot}53795°$ ΔBXO is isosceles $\Rightarrow \angle BOX = 70{\cdot}924°$ $\frac{\sin 70{\cdot}924°}{7{\cdot}675} = \frac{\sin 54{\cdot}53795°}{r}$ $r = 6{\cdot}6145$ $r = 7$	**Scale 5C (0, 2, 4, 5)** *Low Partial Credit:* Cosine rule formulated with some substitution $(31 + r)$ *High Partial Credit:* Relevant equation in r