Question 6 **(25 marks)**

(a) A class group carried out a study of the makes and fuel types of cars in a large carpark.
It found that 30% of the cars ran on diesel and 70% of these diesel cars were Volkswagen.
It found that 60% of the cars ran on petrol and 25% of these petrol cars were Volkswagen.
It found that 10% of the cars were hybrid/electric and 9% of these cars were Volkswagen.
One car is selected at random from the car park.
Find the probability that it is a Volkswagen car.

(b) The Road Safety Authority has data on driving test pass rates at all its test centres.

(i) In a particular Driving Test Centre the probability that a person taking the test for the first time will pass is $\frac{1}{4}$. All of the test results are independent.
In this centre on a particular day Joe, along with 5 others, takes the test.
All six are taking the test for the first time.
Find the probability that Joe passes the test along with exactly 2 others.

(ii) The overall pass rate for all drivers at another centre is $\frac{1}{2}$ (Whether it is their first attempt or a subsequent attempt).
On a particular day, n people take the test in this centre.
The probability that two people or less than two people pass the test can be written in the form

$$\frac{an^2 + bn + c}{2^{n+1}}$$

where $a, b, c \in \mathbb{N}$.
Find the value of a, the value of b, and the value of c.

<table>
<tr><th>Q6</th><th>Model Solution – 25 Marks</th><th>Marking Notes</th></tr>
<tr><td>(a)</td><td>
<table>
<tr><th></th><th>D</th><th>P</th><th>H/E</th></tr>
<tr><td></td><td>0·3</td><td>0·6</td><td>0·1</td></tr>
<tr><td></td><td>$\times 0{\cdot}7$</td><td>$\times 0{\cdot}25$</td><td>$\times 0{\cdot}09$</td></tr>
<tr><td>VW</td><td>0·21</td><td>0·15</td><td>0·009</td></tr>
</table>
$$P(VW) = 0{\cdot}21 + 0{\cdot}15 + 0{\cdot}009$$
$$= 0{\cdot}369$$
</td><td>
Scale 10D (0, 3, 5, 8, 10)
Low Partial Credit:
Any relevant probability from line 1 written

Mid Partial Credit:
Any 1 relevant probability from line 3 formulated or written

High Partial Credit:
All 3 relevant probability from line 3 formulated or written
</td></tr>
<tr><td>(b)
(i)</td><td>
$$\binom{5}{2}\left(\frac{1}{4}\right)^2\left(\frac{3}{4}\right)^3\frac{1}{4} = \frac{135}{2048}$$
</td><td>
Scale 10C (0, 4, 8, 10)
Low Partial Credit:
$\binom{5}{2}$ or $\frac{3}{4}$ or $\left(\frac{1}{4}\right)^2$ or $\left(\frac{1}{4}\right)^3$

High Partial Credit:
$\binom{5}{2}\left(\frac{1}{4}\right)^x\left(\frac{3}{4}\right)^y$ where $x, y \neq 1$
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<tr><td>(b)
(ii)</td><td>
P(2 or less) = P(0 pass + 1 pass + 2 pass)
P(0 pass) = $\left(\frac{1}{2}\right)^n$
P(1 pass) = $\left[\binom{n}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{n-1}\right]$
P(2 pass) = $\left[\binom{n}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{n-2}\right]$
P($\leq$ 2) = $\frac{1}{2^n} + \left[\frac{n}{2^n}\right] + \left[\frac{n(n-1)}{2^{n+1}}\right]$
$$= \frac{2 + 2n + n^2 - n}{2^{n+1}} = \frac{n^2 + n + 2}{2^{n+1}}$$
$$\Longrightarrow\ a = 1,\ \ b = 1,\ \ c = 2.$$
</td><td>
Scale 5C (0, 3, 4, 5)
Low Partial Credit:
P(0 pass + 1 pass + 2 pass)

High Partial Credit:
Any two of $\left(\frac{1}{2}\right)^n$ or $\left[\binom{n}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^{n-1}\right]$ or $\left[\binom{n}{2}\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^{n-2}\right]$
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